

Financial Calculus

Sheldon Lin
sheldon@utstat.utoronto.ca

Mathematical Foundations

- The Binomial Distribution
- The Normal and Lognormal Distributions
- Expectation, Volatility and Laplace Transform
- The Central Limit Theorem and the Law of Large Numbers
- Time Value of Money, Force of Interest, Money Market Account
- Stock Models: Binomial Model and Geometric Brownian Motion
- Actuarial Pricing vs No-Arbitrage Pricing
- Perfect Hedging: Forward Contract as an Example

Discrete Processes and Binomial Trees

- One Period Binomial Model/Tree
- No-Arbitrage Pricing and Risk-Neutral Probabilities
- Random Walk and Binomial Stock Model
- Risk-Neutral Probability Measure
- Pricing Options by Backwards Induction
- Self-Financing Strategy, Perfect Delta Hedging
- Complete Market
- Filtration/Information Structure
- Conditional Expectation with respect to Filtration
- Martingale
- Moving towards Continuous Models

Continuous Processes

- (Arithmetic) Brownian Motion and Properties
- Geometric Brownian Motion Model for Stock Prices
- Ordinary Calculus: Differential and Integral
- Ordinary Calculus: Differential Equations: ODE and PDE

- Stochastic Integral and Properties
- Ito Processes
- Stochastic Calculus: Ito's Lemma
- The Product Rule
- Change of Probability Measure and the Radon-Nikodym Process
- Girsanov Theorem
- From P Probability Measure to Q Probability Measure
- Self-financing Trading Strategy and Replicating Portfolio
- Application to the Black-Scholes Model
- The Black-Scholes PDE

Pricing Market Securities

- Currency Exchange and Exchange Forwards and Options
- Options on Stocks with Dividends
- Non-tradables and Market Price of Risk
- Quantos

Interest Rates

- Zero-Coupon/Discount Bonds
- Yield, Short Rate and Forward Rate
- Forward Rate Model
- SDE for Discount Bonds
- Risk-Neutral Valuation
- The Heath-Jarrow-Morton (HJM) Model
- Short Rate Model
- The Vasicek/Hull-White Model